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SIMPLE METHODS FOR CALCULATING THE AZIMUTHAL COVERAGE OF HF DIRECTIONAL ANTENNAS.

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SIMPLE METHODS FOR CALCULATING THE AZIMUTHAL COVERAGE OF HF DIRECTIONAL ANTENNAS

by

G. May

SUMMARY

Methods of calculating the azimuthal coverage of HF directional antennas are described and worked examples are given. Particular attention is paid to spherical trigonometry formulas employing auxiliary quantities because of their suitability for use with simple hand-held electronic calculators. The relationships between sides and angles of oblique spherical triangles, collated from several sources, are listed and the value of applying tests based on these relationships prior to making detailed solutions of spherical triangle problems is demonstrated.

LIST OF CONTENTS

1   INTRODUCTION

With the increasing use of directional antennas for HF communications on point-to-point and ground-air links the determination of the azimuthal coverage obtained at a given range and bearing from the ground antenna is of considerable interest and importance. Whilst methods of calculating the distance and bearing between two locations on the earth's surface can be found in commonly available mathematical[1,2] and radio engineers' handbooks[3,4], it is apparent that a number of simple aids for resolving the associated spherical triangle problems are not generally known. The reason for this appears to be that these aids are scattered amongst a number of texts. Consequently an attempt has been made to gather these aids together along with the conventional methods and to demonstrate their application and value.

The advent of the cheap electronic hand-held scientific calculator has also meant that several methods, which in the past would have been very time-consuming and tedious to use, can now be employed swiftly, simply and accurately. As an example the use of the haversine formula, which previously would have required access to haversine tables, can now be readily employed since the conversions between haversine and cosine functions can be carried out using a few key strokes on the electronic calculator. It is of interest to note that one of the greatest British mathematicians, John Napier[5] (1550-1617) Eighth Laird of Merchiston, near Edinburgh not only invented logarithms but also Napier's analogies or relationships in spherical trigonometry (see Appendix A). Moreover he was the first to attempt the construction of a calculating machine for the performance of multiplication and division by means of metal plates in a box.

Basic formulas used in solving spherical triangle problems are stated in Appendix A while in Appendix B useful relationships between the sides and angles of oblique spherical triangles are given. In Appendix C the solutions of oblique spherical triangle problems using auxiliary quantities, a method very appropriate for use with electronic calculators, are derived.

The use of the formulas, detailed in Appendices A and C, in the calculation of range and bearing between two locations are discussed in section 2 while detailed calculations and examples are given in Appendix D, where the usefulness of the relationships stated in Appendix B is illustrated.

In section 3 consideration is given to the problems of finding the azimuthal coverage obtained by the use of directive HF antennas and this is amplified by the worked examples of Appendix E.

The choice of units for the calculations of Appendices D and E is not easily made owing to the practical use in military and world-wide airline operations[6] of both Imperial and metric units. The convention has been adopted, therefore, of calculating ranges and distances in nautical miles. A table of conversion factors and constants is given in Appendix F.

A further factor in the writing of this Report has been the preparation of revised material on great circle bearings and distances and new material on the azimuthal

coverage aspects of HF directional antennas for use in the Air Publication[7] dealing with
the general principles of HF and MF antennas for ground stations.

2   CALCULATION OF RANGE AND BEARINGS BETWEEN TWO LOCATIONS WHOSE CO-ORDINATES ARE KNOWN

Two quantities that commonly have to be calculated in planning and deploying HF
antennas are the bearing and distance between two points. Moreover these quantities have
often to be found before other factors, such as the azimuthal coverage given by an
antenna having a known beamwidth, can be calculated.

Fig 1 shows the two locations P and Q both of which will be seen to lie on
meridians through the North Pole. Thus by joining P and Q by the great circle through
these two points, one has only to solve the spherical triangle NPQ in order to determine
the value of PQ and the two angles $N\hat{P}Q$ and $N\hat{Q}P$. The latter enable us to find the
bearings, from true North, of one point to the other, care being taken to remember that
by convention bearings are always taken in a clockwise manner from North (see Appendix D,
section D.1).

Several methods of solving the problems of distance and bearing are demonstrated in
Appendix D and a choice can be made depending upon whether one has access to either a
small pocket electronic calculator or a set of mathematical tables.

It should be noted that in Appendix D all the calculations of great circle distances
and bearings are made on the assumption that the earth is truly spherical. Whilst, as is
pointed out, this assumption is not strictly correct it is nevertheless a reasonable one
for the reasons given. Very accurate calculations of the distance between two widely
separated points on the earth's surface, *ie* the length of the geodesic line, can be made
but it is not a simple matter[8] and is beyond the scope of this Report. Also, in
Appendix D, section D.4 the value of using the relationships between the sides and angles
of the spherical triangle, as listed in Appendix B, is demonstrated. These relationships
help to reduce the uncertainties as to which quadrant and hence which sign of the tri-
gonometrical function should be taken.

3   CALCULATION OF THE AZIMUTHAL COVERAGE OF HF DIRECTIVE ANTENNAS

3.1   Azimuthal coverage parameters

The need to reduce the level of unwanted signals and interference has led to the
use of directional antennas such as rhombics[9], log-periodic[10] and Beverages[11,12], both
for transmission and reception. From the specifications for these directional antennas one
can obtain their azimuthal beamwidth in degrees; typically the half and tenth power beam-
widths are quoted. The problem then arises as to the azimuthal coverage that will be
obtained for the stated performance. Typical questions that arise are as follows:-

(a) Given the beamwidth in degrees what will be the latitude and longitude of
the points at the extremities of that beam at a given range?

(b) What is the latitude of the point where an extremity of the beam crosses a
known meridian?

(c) What is the longitude of the point where the beam extremity crosses a known parallel of latitude?

A diagram showing the main features of these azimuthal coverage problems is shown in Fig 2 and represents a directional antenna assumed to be located at Exeter, point P . The maximum response of the main lobe is directed towards Bodø, point Q , in Northern Norway. The lobe is assumed to be symmetrical and to have a total beamwidth of 20 degrees, *ie* $\pm 10^{\circ}$ from the maximum response along PQ .

Calculations showing how the questions posed in (a), (b) and (c) above may be answered in this particular case are given in Appendix E and are summarized below.

3.2 To find the latitude and longitude of the points at the extremities of the beam at a given range

In Fig 2, J and K are the points where the $\pm 10^{\circ}$ beam extremities cross the line at right angles through the antenna centre-line at Bodø, point Q . It is shown in Appendix E that the range PQ from Exeter to Bodø is 1155 nautical miles and the bearing NP̂Q of Bodø from Exeter is 21.9349°.

The latitude of point J , λ_J , is found from the co-latitude $90 - \lambda_J$, *ie* the arc NJ . The longitude ϕ_J is obtained using the angle PNJ since this is the angle from the meridian through Exeter, whose longitude, ϕ_p , is known.

The calculations of Appendix E show [equations (E-3) and (E-4)] that the co-ordinates of point J are

$$\lambda_J = 69^{\circ}10'N , \quad \phi_J = 06^{\circ}57'E .$$

Similarly from (E-7) and (E-8) the co-ordinates of K are

$$\lambda_K = 65^{\circ}05'N , \quad \phi_K = 20^{\circ}33'E .$$

3.3 To find the latitude of the point where the extremity of the antenna beam crosses a known meridian

In Fig 2 let PJ be the northern limit of the beam which intersects, at point H (latitude λ_H) , the meridian NQ through Bodø . Here we have to find NH the co-latitude of H . Solving for NH using the two angles and included side method, case 2 of Appendix C, it is shown [see equation (E-9)] that NH = 15.8993° and thus $\lambda_H = 90 - NH = 74.1007^{\circ}N = 74^{\circ}06'N$.

3.4 To find the longitude of the point where the extremity of the antenna beam crosses a known parallel of latitude

The antenna beam PJ is assumed to cross the parallel of latitude, λ_D , at point D . The intersection of the meridian NP and the parallel of latitude, λ_D , is at point E (Fig 2).

The longitude, ϕ_D , of D can be obtained if we find the angle PN̂D which is the difference of longitudes of P and D . In Appendix E it is shown, equation (E-14),

that

$$\tan \hat{PND} = \frac{\sin(\lambda_D - \lambda_P) \tan \hat{DPN}}{\cos \lambda_D}$$

and the value of $\hat{PND}$ is 8.7854° .

Hence

$$\phi_D = \phi_P - \hat{PND} = 4.25^{\circ}W - \hat{PND} = 4.5354^{\circ}E = 04^{\circ}32'E .$$

4   CONCLUSIONS

Various approaches to obtaining great circle distances and bearings have been examined and the advantage of the auxiliary-quantities method when using an electronic calculator has been shown. The methods of solving the spherical triangles that occur in finding the azimuthal coverage of HF directional antennas have been discussed and the value of examining the relationships between the sides and angles of the triangles, prior to making detailed calculations, has been demonstrated.

Appendix A

FORMULAS FOR SOLVING SPHERICAL TRIANGLE PROBLEMS

Adopting the usual conventions the sides (arcs) are denoted by a, b, c and the angles opposite them by α, β and γ respectively (see Fig 1).

Law of sines

$$\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma} \; . \tag{A-1}$$

Law of cosines for sides

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha \tag{A-2}$$

$$\cos b = \cos c \cos a + \sin c \sin a \cos \beta \tag{A-3}$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma \; . \tag{A-4}$$

Law of cosines for angles

$$\cos \alpha = - \cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a \tag{A-5}$$

$$\cos \beta = - \cos \gamma \cos \alpha + \sin \gamma \sin \alpha \cos b \tag{A-6}$$

$$\cos \gamma = - \cos \alpha \cos \beta + \sin \alpha \sin \beta \cos c \; . \tag{A-7}$$

Napier's analogies

$$\tan \frac{\alpha - \beta}{2} = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}(a + b)} \cot \frac{\gamma}{2} \tag{A-8}$$

$$\tan \frac{\alpha + \beta}{2} = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} \cot \frac{\gamma}{2} \tag{A-9}$$

$$\tan \frac{a - b}{2} = \frac{\sin \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2}(\alpha + \beta)} \tan \frac{c}{2} \tag{A-10}$$

$$\tan \frac{a + b}{2} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\cos \frac{1}{2}(\alpha + \beta)} \tan \frac{c}{2} \tag{A-11}$$

other forms can be obtained by cyclic changing of the letters.

Gauss' or Delambre's analogies

$$\sin \frac{\alpha - \beta}{2} = \frac{\sin \frac{1}{2}(a - b)}{\sin c/2} \cos \frac{\gamma}{2} \tag{A-12}$$

$$\sin \frac{\alpha + \beta}{2} = \frac{\cos \frac{1}{2}(a - b)}{\cos c/2} \cos \frac{\gamma}{2} \qquad \text{(A-13)}$$

$$\cos \frac{\alpha - \beta}{2} = \frac{\sin \frac{1}{2}(a + b)}{\sin c/2} \sin \frac{\gamma}{2} \qquad \text{(A-14)}$$

$$\cos \frac{\alpha + \beta}{2} = \frac{\cos \frac{1}{2}(a + b)}{\cos c/2} \sin \frac{\gamma}{2} . \qquad \text{(A-15)}$$

Again other forms can be obtained by cyclic changing of the letters.

Haversine formulas[13] for the sides

$$\text{hav } a = \text{hav}(b - c) + \sin b \sin c \text{ hav } \alpha \qquad \text{(A-16)}$$

$$\text{hav } b = \text{hav}(c - a) + \sin c \sin a \text{ hav } \beta \qquad \text{(A-17)}$$

$$\text{hav } c = \text{hav}(a - b) + \sin a \sin b \text{ hav } \gamma \quad , \qquad \text{(A-18)}$$

where hav θ is the haversine of an angle θ and is defined by

$$\text{hav } \theta = \tfrac{1}{2}(1 - \cos \theta) = \sin^2 \theta/2 \qquad \text{(A-19)}$$

from (A-19)

$$\cos \theta = 1 - 2 \text{ hav } \theta \qquad \text{(A-20)}$$

and

$$\theta = \text{arc } \cos(1 - 2 \text{ hav } \theta) . \qquad \text{(A-21)}$$

Haversine formulas[13] for the angles

$$\text{hav } \alpha = \sin(s - b) \sin(s - c) \text{ cosec } b \text{ cosec } c \qquad \text{(A-22)}$$

$$\text{hav } \beta = \sin(s - c) \sin(s - a) \text{ cosec } c \text{ cosec } a \qquad \text{(A-23)}$$

$$\text{hav } \gamma = \sin(s - a) \sin(s - b) \text{ cosec } a \text{ cosec } b \qquad \text{(A-24)}$$

where $s = \tfrac{1}{2}(a + b + c)$.

Appendix B

RELATIONSHIPS BETWEEN THE SIDES AND ANGLES OF OBLIQUE SPHERICAL TRIANGLES

In solving problems associated with oblique spherical triangles the following relationships[1,13,14] between the sides and angles are found to be of considerable value. It should be noted that they are restricted to those triangles in which each side and angle is less than 180°.

B1   The sum of any two sides is greater than the third.

B2   The sum of the three sides is less than 360°.

B3   If two sides are equal, the angles opposite are equal and conversely.

B4   The sum of the three angles is greater than 180° and less than 540°.

B5   Half the sum of two sides and half the sum of the two angles opposite them belong to the same quadrant.

B6   If a side (or an angle) differs from 90° by a larger number of degrees than another side (or angle) then the first side and its opposite angle belong to the same quadrant.

Appendix C

SOLUTION OF OBLIQUE SPHERICAL TRIANGLE PROBLEMS USING AUXILIARY QUANTITIES

In many problems it is required to find only one or two of the unknown parts of a spherical triangle. It is evident from Appendix A that any part can be found using any three other parts through the use of the general formulas given there. By the introduction of auxiliary quantities[14] the basic sine and cosine formulas have been adapted to logarithmic computation and are therefore in a form that can be easily processed on an electronic calculator. As these methods may not be generally known a full statement will be given of the derivation for one case, two sides and the included angle. Formulas for the remaining cases which can be derived in a similar fashion are also given. Again the usual conventions are employed (see Appendix A and Fig 1).

Case 1. **Two sides and the included angle**

Let a, b and γ be the given parts.

To find c from (A-4) we have that the relation between a, b, γ and c is

$$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma . \qquad \text{(C-1)}$$

Let

$$m \sin M = \sin b \cos \gamma \qquad \text{(C-2)}$$

and

$$m \cos M = \cos b \qquad \text{(C-3)}$$

from (C-1), (C-2) and (C-3) by eliminating b

$$\cos c = m \cos(a - M) . \qquad \text{(C-4)}$$

From (C-2) and (C-3)

$$\tan M = \tan b \cos \gamma \qquad \text{(C-5)}$$

and from (C-3) and (C-4)

$$\cos c = \frac{\cos b \cos(a - M)}{\cos M} . \qquad \text{(C-6)}$$

To find β Similarly using the relationship between a, b, γ and β it can be shown[14] that

$$\cot \beta = \frac{\cot b \sin a - \cos a \cos \gamma}{\sin \gamma}$$

and again using (C-5) it can also be shown that

$$\tan \beta = \frac{\tan \gamma \sin M}{\sin(a - M)} . \qquad \text{(C-7)}$$

<u>To find</u> α Interchanging a and b and consequently α and β in (C-5) and (C-7) and calling the auxiliary angle N we have

$$\tan N = \tan a \cos \gamma \qquad (C-8)$$

and

$$\tan \alpha = \frac{\tan \gamma \sin N}{\sin(b - N)} \qquad (C-9)$$

cos c also can be found using N as an auxiliary angle

$$\cos c = \frac{\cos a \cos(b - N)}{\cos N} . \qquad (C-10)$$

Case 2.   <u>Given two angles and the included side</u>

Let α, β and c be the given parts.

To find γ and a we have

$$\tan M = \frac{1}{\tan \alpha \cos c} \qquad (C-11)$$

then

$$\cos \gamma = \frac{\cos \alpha \sin(\beta - M)}{\sin M} \qquad (C-12)$$

and

$$\tan a = \frac{\tan c \cos M}{\cos(\beta - M)} . \qquad (C-13)$$

To find b we have

$$\tan N = \frac{1}{\tan \beta \cos c} \qquad (C-14)$$

and

$$\tan b = \frac{\tan c \cos N}{\cos(\alpha - N)} . \qquad (C-15)$$

Case 3.   <u>Given two sides and an angle opposite one of them</u>

Let a, b and α be the given parts.

To find c using

$$\tan M = \tan b \cos \alpha \qquad (C-16)$$

$$\cos(c - M) = \frac{\cos a \cos M}{\cos b} . \qquad (C-17)$$

To find γ and β using

$$\tan N = \tan \alpha \cos b \qquad (C-18)$$

$$\sin(\gamma + N) \;=\; \frac{\sin N \,\tan b}{\tan a} \tag{C-19}$$

and

$$\sin \beta \;=\; \frac{\sin b \,\sin \alpha}{\sin a} \;. \tag{C-20}$$

Case 4. Given two angles and a side opposite one of them

Let α, β and a be the given parts.

To find γ

$$\tan M \;=\; \frac{1}{\tan \beta \,\cos a} \tag{C-21}$$

then

$$\sin(\gamma - M) \;=\; \frac{\cos \alpha \,\sin M}{\cos \beta} \;. \tag{C-22}$$

To find c

$$\tan N \;=\; \tan a \,\cos \beta \tag{C-23}$$

then

$$\sin(C - N) \;=\; \frac{\tan \beta \,\sin N}{\tan \alpha} \;. \tag{C-24}$$

To find b

$$\sin b \;=\; \frac{\sin \beta \,\sin a}{\sin \alpha} \;. \tag{C-25}$$

Appendix D

DETAILED CALCULATIONS OF GREAT CIRCLE DISTANCES AND BEARINGS BETWEEN TWO POINTS WHOSE CO-ORDINATES ARE GIVEN

D.1 General principles

It is desired to determine the great circle distance and bearing between the two locations P and Q depicted in Fig 1. It will be seen that the great circle arcs NP and NQ are part of the meridians through P and Q . Let the latitudes of P and Q be denoted by λ_P, λ_Q and the longitudes by ϕ_P and ϕ_Q . In the triangle NPQ, NP = b = $90 \pm \lambda_P$ and NQ = a = $90 \pm \lambda_Q$ depending whether P and Q are south or north of the equator. The angle $P\hat{N}Q$ designated γ , is derived from the difference in longitudes of P and Q . However, one must always take the minor arc, thus $\gamma = \phi_P - \theta_Q$ or $360 - (\phi_P - \phi_Q)$ depending upon whether $(\phi_P - \phi_Q)$ is less or greater than 180° .

To find the great circle distance between P and Q we have to calculate the value of the arc PQ in degrees, *ie* c , and then multiply by 60 to obtain the answer in nautical miles. This can also be stated in kilometres using the internationally agreed conversion factor[15] of 1 NM = 1.852 km. The assumption is made in using these conversions, that the earth is truly spherical. Whilst it has recently been determined that the earth is slightly pear-shaped[16] the variations are relatively small (see Appendix E) and are usually neglected.

The value of PQ can be determined since we know the values of NP, NQ (b and a respectively) and the included angle $P\hat{N}Q$ (*ie* γ). Similarly angle $N\hat{P}Q$ designated α , or $N\hat{Q}P$ designated β may be determined. By convention true relative bearings are always taken in a clockwise direction from North. Hence though β is the true bearing of P relative to Q , the true bearing of Q relative to P is $360 - \alpha$.

D.2 Use of cosine and sine laws

From equation (A-4) we have

$$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma$$

but $a = 90 \pm \lambda_Q$, $b = 90 \pm \lambda_P$, $\gamma = \phi_P - \phi_Q$ or $360 - (\phi_P - \phi_Q)$. When λ_P and λ_Q are both north or both south of the equator then

$$\cos c = \sin \lambda_P \sin \lambda_Q + \cos \lambda_P \cos \lambda_Q \cos \gamma . \qquad (D-1)$$

When λ_P is north and λ_Q is south of the equator or vice versa

$$\cos c = - (\sin \lambda_P \sin \lambda_Q) + \cos \lambda_P \cos \lambda_Q \cos \gamma . \qquad (D-2)$$

As an example find the distance from Seattle, USA to Sydney, Australia

	Latitude	Longitude
Seattle (point P)	$47^\circ27'$N	$122^\circ21'$W
Sydney (point Q)	$33^\circ54'$S	$151^\circ12'$E

The difference in longitude is $273°33'$ but this is the major arc whereas as noted in section D.1, γ is taken as the value of the minor arc so

$$\gamma = 360° - 273°33' = 86°27' .$$

Substituting for λ_P, λ_Q and γ in equation (D-1) and evaluating we find

$$\cos c = - 0.3761 \quad \text{and} \quad c = 112.0941°$$

thus the distance PQ is 6726 NM.

D.3 Use of haversine formula

A great advantage of the haversine method is that there is a single positive angle less than $180°$ having a given haversine value, thus eliminating the uncertainty as to which quadrant an angle or side belongs.

Using equation (A-18) and decimalising the values for Seattle and Sydney quoted in section D.2

$$a = 123.9° , \quad b = 42.55° , \quad \gamma = 86.45°$$

$$\begin{aligned} \text{hav } C &= \text{hav}(a - b) + \sin a \sin b \text{ hav } \gamma \\ &= \text{hav } 81.35° + \sin 123.9° \sin 42.55° \text{ hav } 86.45°. \end{aligned} \tag{D-3}$$

Using (A-19) to find the haversine values

$$\begin{aligned} \text{hav } 81.35° &= 0.424\ 801 \\ \text{hav } 86.45° &= 0.469\ 040. \end{aligned}$$

Thus (D-3) becomes

$$\begin{aligned} \text{hav } c &= 0.424\ 801 + 0.263\ 264 \\ &= 0.688\ 065 . \end{aligned}$$

Using equation (A-20)

$$\cos c = - 0.376\ 130$$

$$c = 112.094\ 15°$$

and the distance is 6726 NM.

D.4 Use of auxiliary quantities to find the distance and bearing of one location from the other

In many applications one has to find only the bearing from one's location to some other location, and it is here that the method described in Appendix C where an auxiliary angle is used, proves to be especially useful.

Frequently one has to find both the distance and the bearing between two points but still only one auxiliary angle need be calculated. Before proceeding with the calculation

it is advisable to ascertain which quadrant the required angle will be in thus avoiding
the uncertainties that can arise. The relationships between sides and angles of an
oblique spherical triangle are stated in Appendix B and an application of these will now
be shown using the Seattle-Sydney co-ordinates again.

We know from (D-1) that we have to find the third side c and either angle α or
angle β (Fig 1) if we wish to find the distance and the bearing between the two points
P and Q . We can, therefore, use equation (C-8) to find the auxiliary angle N and
equations (C-9) and (C-10) to find both the bearing and distance, respectively, of
Sydney from Seattle knowing that

$$Q = 123.90^{\circ}, \quad b = 42.55^{c}, \quad \gamma = 86.45^{\circ}.$$

Applying the tests in Appendix B we can deduce the following:- from B6 we see that
$90 - b > a - 90$ but b is in the first quadrant hence β also is in the first quadrant.
It then follows that $\frac{1}{2}(\beta + \gamma) < 90^{\circ}$ so from B5 $\frac{1}{2}(b + c) < 90$ and thus $c < 137.45^{\circ}$.

From B1 $(b + c) > a$, *ie* $c > a - b$ thus $c > 81.35^{\circ}$ consequently $81.35^{\circ} < c < 137.45^{\circ}$.
We can now see that $\frac{1}{2}(a+c)$ must be in the second quadrant and hence again from B5
$\frac{1}{2}(\alpha + \gamma)$ is also in the second quadrant thus $180^{\circ} < (\alpha + \gamma) < 360^{\circ}$.
Now

$$\gamma = 86.45^{\circ} \quad so \quad 93.55^{\circ} < \alpha < 273.55^{\circ}.$$

Knowing also that b and β are in the first quadrant and applying the sine rule we
see that in order that $\sin \alpha$ may remain positive, α must be in the second quadrant.
Since a and α are in the same quadrant, the second, then $a - 90 > c - 90$
thus $a > c$, *ie* $c < 123.9^{\circ}$.

Summarizing we have

$$93.55^{\circ} < \alpha < 180^{\circ}$$
$$81.35^{\circ} < c < 123.9^{\circ}.$$

Now from (C-8)

$$\tan N = \tan a \cos \gamma$$
$$= \tan 123.90^{\circ} \cos 86.45^{\circ}$$

and

$$N = -5.2647^{\circ}$$

substituting in (C-10)

$$\cos c = \frac{\cos a \cos(b - N)}{\cos N}$$
$$= \frac{\cos 123.90^{\circ} \cos(42.55^{\circ} + 5.2647^{\circ})}{\cos(-5.2647^{\circ})}$$

and

$$c = 112.0941^{\circ}.$$

From (C-9)

$$\tan \alpha \quad = \quad \frac{\tan \gamma \sin N}{\sin(b - N)}$$

and substituting we have

$$\tan \alpha \quad = \quad \frac{\tan 86.45^{\circ} \sin(- 5.2647^{\circ})}{\sin(42.55^{\circ} + 5.2647^{\circ})}$$

$$\alpha \quad = \quad 116.6103^{\circ} \quad \approx \quad 116^{\circ}37' \ .$$

Thus as noted in (D-1) the true bearing of Sydney from Seattle, by convention taken as clockwise from North, is

$$360 - \alpha \quad = \quad 243.3897^{\circ} \quad = \quad 243^{\circ}23' \ .$$

Appendix E

DETAILED CALCULATIONS OF AZIMUTHAL COVERAGE OF AN ANTENNA AT EXETER DIRECTED TO BODØ

E.1 Geometry of the situation

The geometry of the situation is depicted in Fig 2 where Exeter is at point P and
Bodø is at point Q . The latitudes and longitudes of Exeter and Bodø are as follows:-

	Latitude	Longitude
Exeter	50.40°N	4.25°W
Bodø	67.27°N	14.33°E .

Using one of the methods described in Appendices C or D the distance along the great
circle PQ is found to be 19.2431°, ie 1155 NM and the bearing of Bodø from Exeter,
$\hat{NPQ}$, 21.9349° from true North.

The half-power beamwidth is assumed to be 20° thus $\hat{JPQ} = \hat{KPQ} = 10^{\circ}$
where points J and K are on the line JK at right angles to the great circle PQ
and passing through Bodø, point Q .

Thus

$$\hat{JOP} = \hat{KQP} = 90^{\circ} .$$

Since $\hat{NPQ} = 21.9349^{\circ}$, $\hat{NPJ} = 11.9349^{\circ}$ and $\hat{NPK} = 31.9349^{\circ}$.
Also NP = co-latitude of P = $90^{\circ} - 50.40^{\circ}$ = 39.60°.

E.2 To find the latitude and longitude of points at the extremities of the beam at a given range

For this example we will take the points J and K (see Fig 2).

(a) Point J

In the spherical triangle JPQ we know the values of $\hat{JPQ}$, $\hat{JQP}$ and PQ so
we can now use equations (C-14) and (C-15). Noting that auxiliary angle N = 0° since
tan $\hat{JQP}$ = tan 90° = ∞ , we find tan PJ = tan PQ/cos $\hat{JPQ}$.

·This expression could also have been found from the tables of formulas for right
spherical triangles[1,2,3,13].

Thus

$$\tan PJ = \tan 19.2431^{\circ}/\cos 10^{\circ}$$

so

$$PJ = 19.5177^{\circ} , \quad ie\ 1171\ NM . \tag{E-1}$$

Also

$$\hat{NPJ} = 11.9349^{\circ} . \tag{E-2}$$

In triangle NPJ we now know the values of NP, PJ and $\hat{NPJ}$, thus using
equation (C-5) we find the auxiliary angle M to be 19.1267° and we solve, using
equations (C-6) and (C-7) to find NJ and $\hat{PNJ}$.

From equation (D-6) NJ = 20.8369°.

From equation (D-7) PNJ = 11.2001°.

Now NJ is the co-latitude of J and thus the latitude of J is

$$\lambda_J = 90^{\circ} - NJ = 69.1631^{\circ}N = 69^{\circ}10'N . \tag{E-3}$$

The longitude of J, ϕ_J, is 11.2001° east of the meridian through Exeter which is at 04.25°W, *ie*

$$\phi_J = 4.25^{\circ}W - 11.2001^{\circ} = 6.9501^{\circ}E = 06^{\circ}57'E . \tag{E-4}$$

(b) Point K

PK may be calculated in the same manner as for PJ . However, if PJ has been calculated there is strictly no need to calculate PK as well, since by symmetry they are equal, thus

$$PK = 19.5177^{\circ} , \quad ie \; 1171 \; NM . \tag{E-5}$$

In triangle NPK we know, therefore, the values of NP, PK and NPK and using equation (C-5) we find the auxiliary angle, M ,

$$M = 19.1267^{\circ} . \tag{E-6}$$

From equation (C-6)

$$NK = 24.9103^{\circ}$$

$$\therefore \; \lambda_K = 90 - NK = 65.0897^{\circ}N = 65^{\circ}05'N . \tag{E-7}$$

From equation (C-7)

$$P\hat{N}K = 24.8075^{\circ} .$$

Therefore the longitude of K , ϕ_K , is 24.8075° east of the meridian through Exeter, *ie*

$$\phi_K = 04.25^{\circ}W - 24.8075^{\circ} = 20.5575^{\circ}E = 20^{\circ}33'E . \tag{E-8}$$

E.3 **To find the latitude of the point where the extremity of the antenna beam crosses a known meridian**

Here we will consider the case where the northern extremity of the beam crosses at H , the meridian through Bodφ. To obtain the latitude, λ_H , of point H we need to find the co-latitude NH .

In the spherical triangle HPN we know the values of the two angles $H\hat{P}N$ and $H\hat{N}P$ and the included side NP since $H\hat{P}N = N\hat{P}J = 11.9349^{\circ}$ see equation (E-2) and $H\hat{N}P = P\hat{N}Q = \phi_P - \phi_Q = 18.58^{\circ}$ also NP the co-latitude of P = 39.60°. Thus we can calculate the value of NH .

Using the auxiliary angle method of Appendix C, equation (C-11)

$$\tan M = \frac{1}{\tan H\hat{P}N \cos NP} = \frac{1}{\tan 11.9349^{\circ} \cos 39^{\circ}}$$

and

$$M = 80.7498^{\circ} .$$

From (C-13)

$$\tan NH = \frac{\tan NP \cos M}{\cos(H\hat{N}P - M)}$$

and

$$NH = 15.8993^{\circ} .$$

$$\therefore \quad \lambda_H = 90^{\circ} - NH = 74.1007^{\circ}N = 74^{\circ}06'N . \qquad (E-9)$$

If we calculate the values of $N\hat{H}P$ and HP using equations (C-12), (C-14), (C-15) but first apply test B4 from Appendix B which shows that $N\hat{H}P > 149.485^{\circ}$, we find

$$N\hat{H}P = 151.2371^{\circ} \quad \text{and} \quad HP = 24.9663^{\circ} .$$

Applying the sine rule as a check we obtain

$$\frac{\sin 15.8993^{\circ}}{\sin 11.9349^{\circ}} = \frac{\sin 39.60^{\circ}}{\sin 151.2371^{\circ}} = \frac{\sin 24.9663^{\circ}}{\sin 18.58^{\circ}} = 1.324693$$

thus confirming the validity of the calculations.

E.4 To find the longitude of the point where the extremity of the antenna beam crosses a known parallel of latitude

The northern extremity of the antenna beam PJ will again be used and its point of intersection, D , will be assumed to be on the 67th parallel of latitude. Denote the point where the 67th parallel crosses the meridian, ϕ_P , through Exeter, by E . If we then find $E\hat{N}D \equiv P\hat{N}D$ and noting that Exeter is west of the Greenwich meridian, it will be seen that the longitude ϕ_D of point D is given by

$$\phi_D = \phi_P - PND . \qquad (E-10)$$

From the sine formula (A-1) when $\gamma = 90^{\circ}$ we obtain

$$\sin \alpha = \frac{\sin a}{\sin c} \quad \text{and} \quad \sin \beta = \frac{\sin b}{\sin c}$$

also from the cosine formula (A-4) when $\gamma = 90^{\circ}$

$$\cos c = \cos a \cos b$$

combining the above expressions we can show that

$$\sin a = \tan b \cot \beta . \qquad (E-11)$$

Then in triangle DEN where $D\hat{E}N = 90^o$ and $EN = 90 - \lambda_D$, applying (E-11)

$$\sin(90 - \lambda_D) = \tan ED \cot D\hat{N}E$$

and rearranging,

$$\cos \lambda_D \tan D\hat{N}E = \tan ED . \qquad (E-12)$$

Similarly in triangle DEP ,

$$D\hat{E}P = 90^o \quad \text{and} \quad EP = \lambda_D - \lambda_P .$$

Again using (E-11),

$$\sin(\lambda_D - \lambda_P) \tan D\hat{P}E = \tan ED , \qquad (E-13)$$

equating (E-12) and (E-13) and substituting $D\hat{P}N$ for $D\hat{P}E$ since they are identical, we have

$$\tan P\hat{N}D \equiv \tan D\hat{N}E = \frac{\sin(\lambda_D - \lambda_P) \tan D\hat{P}N}{\cos \lambda_D} . \qquad (E-14)$$

Now

$$\lambda_D = 67^o , \quad \lambda_P = 50.4^o \quad \text{and} \quad D\hat{P}N = 11.935^o .$$

$$\therefore \tan P\hat{N}D = \frac{\sin 16.6^o \tan 11.935^o}{\cos 67^o}$$

and

$$P\hat{N}D = 8.7854^o \qquad (E-15)$$

substituting (E-15) in (E-10)

$$\phi_D = \phi_P - P\hat{N}D = 04.25^o W - 8.7854^o = 4.5354^o E$$

$$= 04^o 32' E . \qquad (E-16)$$

Appendix F

CONVERSION FACTORS AND CONSTANTS

F.1 <u>Conversion factors</u>[15]

1 international nautical mile = 1852 m

1 metre = 3.280 84 ft

F.2 <u>Constants</u>[17]

Earth's equatorial radius 6378.160 km = 3443.845 NM

        polar   radius 6356.775 km = 3432.384 NM

        mean   radius 6371.020 km = 3440.076 NM.

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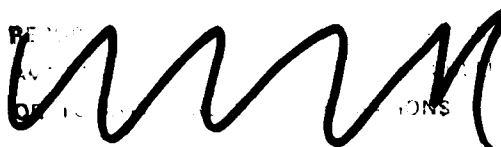| No.	Author	Title, etc
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Fig 1

Fig 1 Spherical triangle NPQ

Fig 2

Fig 2   Diagram for HF directional antenna azimuthal coverage problems
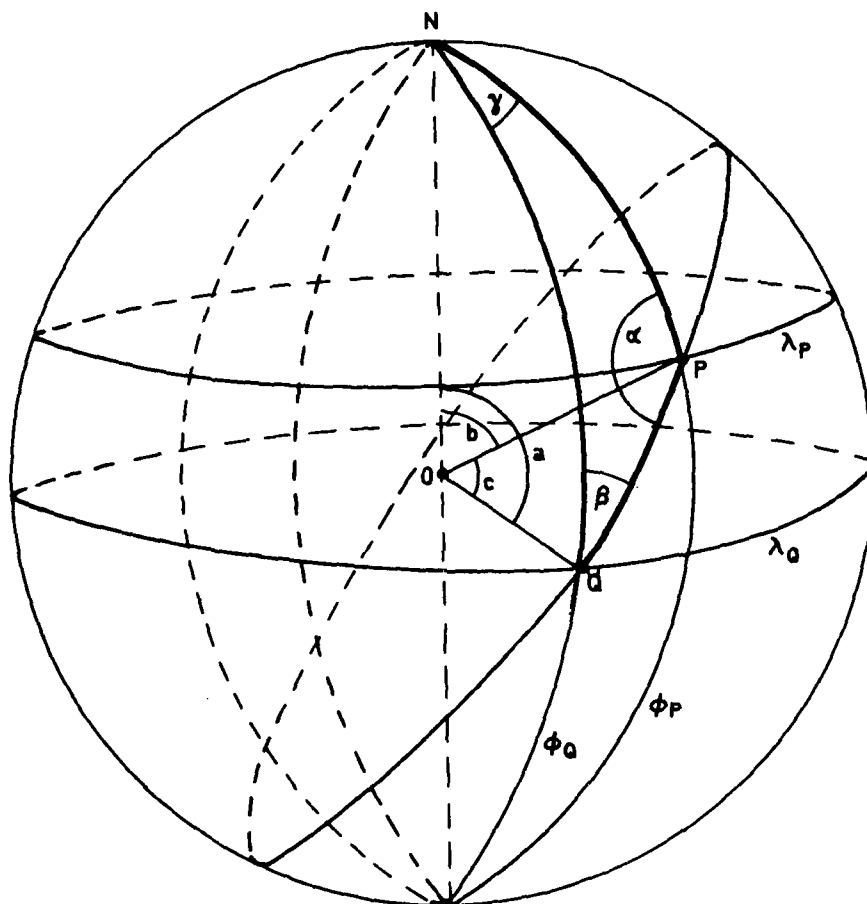
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17. Abstract

   Methods of calculating the azimuthal coverage of HF directional antennas are described and worked examples are given. Particular attention is paid to spherical trigonometry formulas employing auxiliary quantities because of their suitability for use with simple hand-held electronic calculators. The relationships between sides and angles of oblique spherical triangles, collated from several sources, are listed and the value of applying tests based on these relationships prior to making detailed solutions of spherical triangle problems is demonstrated.

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